MESSAGE

16 Hard Arithmetic Isn't Deep Mathematics

TEACHING MORE THAN COMPUTATION

The kind of mathematics that adults need today goes far beyond what once was sufficient. In the past, it might have been enough to know how to read, write, and do the basic measurement and arithmetic they might use in everyday life. In the past, it might also have been enough for students who were going to college to master a set of algebraic tools that enabled them to enroll in higher-level mathematics or science courses. But today's world is characterized by rapid change and pervasive technology. More of our students are likely to participate in some kind of postsecondary education than ever before, and the jobs in demand now didn't even exist five years ago.

In this environment, how do we raise the bar on the mathematical proficiency that we expect of all students, regardless of what path they might pursue after high school? What kind of mathematics do all students need? And how likely is it that all students can achieve the goals that we set?

The Meaning of Depth

We can begin to raise the bar on mathematical proficiency by eliminating some of the endless repetition in our curriculum. We can choose to focus on fewer topics at each grade level, distributing important mathematical ideas across the grades rather than superficially teaching a little bit about many topics in each grade. In this way, students have a chance to learn mathematics deeply, so that they don't need to be taught the same topics year after year. Depth means that students know a lot about multiplication before they deal with an algorithm for performing multiplication. Depth means that when we introduce fractions, we teach students what fractions represent, in what kinds of situations fractions might be useful, how fractions compare to one another, how they relate to what students know about whole numbers, what it means when the numerator or

denominator increases or decreases, and so on. Depth means that before students confront the rules for operating with fractions—what students remember as going straight across, turning upside down, or finding a common denominator—we ensure that they know a lot about fractions and a lot about operations. Depth means that when students study ratios, they go well beyond solving proportions to recognize and utilize proportional relationships in many apparently different settings (like scale drawings, percent increase, sales tax, maps, multiplicative growth and even early notions of mathematical functions), in the process helping them connect mathematical ideas from prekindergarten through grade 12. Depth also means that students earning credit for a high school algebra course not only know how to solve equations but also have built a strong base of algebraic thinking and have developed a diverse set of algebraic tools with which they can represent, model, and solve many kinds of problems—both within and outside of mathematics.

WHAT DEPTH IS NOT

Depth does not mean making all students master arithmetic procedures faster, earlier, or with more digits. A school system whose standards include the mastery of fraction operations earlier than the standards of another system does not necessarily have a more rigorous curriculum. Depth does not mean narrowing our curriculum to numbers and operations alone at the expense of measurement, geometry, and data analysis, where those numbers and operations are actually used and where students learn critical non-numerical ways of thinking as well. Depth does not necessarily mean more exercises. Focusing on more or longer arithmetic procedures at the expense of deeper explorations and problem solving is not the same as raising our expectations for all students. And depth does not have to be painful or boring.

WHERE DEPTH IS SHALLOW

Some of the students least likely to experience deep mathematics are those identified as members of special populations. Students whose first language is not English, students who have learning disabilities, students with physical limitations, students in special education programs, and students who lag behind for whatever reason too often receive the narrowest mathematics teaching we have to offer, usually limited to numerical facts and procedures. Often, compassionate teachers determine that students who have difficulty dealing with language should be limited to working on strictly numerical exercises involving no words. Consequently, many of these students never see a challenging problem or an engaging context simply because their teachers have determined that it would be too frustrating for them. We cannot be surprised when such students perform poorly on tests of higher-level thinking if their experience has never prepared them to think. These students, especially, need strong support and nurturing that goes well beyond arithmetic, even hard arithmetic.

What Can We Do?

While visiting schools, I have found wonderful examples of classrooms where students are learning mathematics in depth. In these classrooms, students experience mathematics taught with strong understanding. They are actively engaged in meaningful and challenging mathematics that may include but go well beyond arithmetic. In these schools, the door is open for every student to meet rigorous mathematics expectations, and students are naturally more engaged in the problems they see and the mathematics they learn.

Depth is not the same as hard arithmetic. Depth comes when students get it. This means that students need to see the contexts in which mathematical ideas arise, need to wrestle with those ideas in problems that take some time to solve, and need opportunities to represent and communicate what they learn.

If we define our mathematics curriculum—the standards that states adopt and test—in ways that focus on students knowing, understanding, and using mathematics, and not just doing hard arithmetic, we can achieve this depth. If we also make shifts in how we structure our classrooms and engage our students, we can ensure that all students can achieve the high goals we set as we raise the bar on what it means to know and be able to use mathematics well.

Reflection and Discussion

FOR TEACHERS

- What issues or challenges does this message raise for you? In what ways do you agree with or disagree with the main points of the message?
- If your state or school system is shifting its curriculum and standards toward deep mathematics rather than hard arithmetic, how can your teaching support this shift? If the curriculum and standards are not changing, in what ways can you help your students make a shift toward deep mathematics?

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